MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

BNA3014 –NUMERICAL ANALYSIS

(All sections / Groups)

20 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **4 pages** including cover page and formulae.

2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.

Question 1 (12.5 marks)

a) (i) Approximate the polynomial $P(x) = x^3 - 3x^2 + 3x - 1$ at $x = 2.72$. Use 4-digit rounding arithmetic. **[1 mark]**

(ii) Rewrite the polynomial $Q(x) = x^3 - 3x^2 + 3x - 1$ in **nested form** and approximate at $x = 2.72$. Use 4-digit rounding arithmetic. **[2 marks]**

(iii) Compare the answer in (i) and (ii) with the true values, $P(x) = Q(x) = 5.088448$. Find their relative errors. Which polynomial approximation has less error? **[3 marks]**

b) Find the root of the following equation by using **Secant** method with tolerance 1×10^{-3}.

$$f(x) = e^x + x - 7, \quad \text{starting with } a = 1,\ b = 2$$

Note: Write your answer correct to 4 decimal places. **[6.5 marks]**

Question 2 (12.5 marks)

a) Given the system $Ax = b$ where

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(i) When $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$, $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$ and $A=LU$, find the lower-triangular matrix, L and upper-triangular matrix, U. **[5.5 marks]**

(ii) Using LU Decomposition, solve the system $Ax = b$. **[4 marks]**

b) Use **Lagrange interpolating polynomial** to find the third degree order polynomial, $P_3(x)$ on the basis of the data given below:

$x_0 = 1$; $f(x_0) = 3$
$x_1 = 2$, $f(x_1) = 6$
$x_2 = 3$, $f(x_2) = 19$
$x_3 = 5$, $f(x_2) = 99$

Hence, evaluate $P_3(4)$. **[3 marks]**

Continued...

Question 3 (12.5 marks)

a) Solve the following set of equations by using **Gauss Seidel** iteration with tolerance 1×10^{-3} (start with $x = 0, y = 0, z = 0$).

$$x - 8y - 2z = 1$$
$$x + y + 5z = 4$$
$$3x - y + z = -2$$

Note: Write your answer correct to 5 decimal places.

[6 marks]

b) Consider a function $f(x)$ such that $f(-2) = 8, f(0) = 4, f(1) = 2, f(3) = -2$.

(i) Construct a **divided difference** table.
(ii) Estimate $f(0.5)$ using a **first degree** interpolating polynomial.
(iii) Estimate $f(0.5)$ using a **second degree** interpolating polynomial.
(iv) Estimate $f(0.5)$ using a **third degree** interpolating polynomial.
(v) Is there any **advantage** here in using second and third degree polynomial?

[6.5 marks]

Question 4 (12.5 marks)

a) The following are values of heat capacity, y at various temperature x for a gas:

x	-50	-30	0	60	90	110
y	1270	1280	1350	1480	1580	1700

Obtain a model that represents the above data by using method of least square. Fit the model equation of the form $\boldsymbol{y = ae^{bx}}$

[6 marks]

b) Given the following differential equation:

$y' = 4x^3 + 12x^2 - 20x + 10$ for $0 \leq x < 1$ with initial condition at $x = 0, y = 1$ and $h = 0.25$.

Compute the value of y_1 and y_2 (6 decimal places) by using each of the following method:

(i) **Euler's Method**
(ii) **Runge-Kutta Second Order Method**

[6.5 marks]

End of Page

FORMULAE

Taylor Polynomial

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

Lagrange Polynomial

$$P_n(x) = \sum_{i=0}^{n} f(x_i) L_i(x)$$

Newton's Polynomial

$$P_n(x) = a_0 + a_1(x - x_0) + \ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

The Secant Method

$$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$$

Euler Method

$$y_{n+1} = y_n + hf(x_n, y_n); \qquad n = 0,1,2,3,\cdots .$$

Runge-Kutta second order methods

$$k_1 = f(x_n, \quad y_n)$$

$$k_2 = f(x_n + h, \quad y_n + hk_1)$$

$$y_{n+1} - y_n + \frac{h}{2}(k_1 + k_2)$$

Exponential Regression

$$y = ae^{bx}$$

$$\sum z = nA + b\sum x$$

$$\sum xz = A\sum x + b\sum x^2$$

$$A = \ln a$$